

Chemical freeze-out parameters at RHIC from the microscopic model calculations

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L.V. Bravina et al., J. Phys. G25 (1999) 351.

Phys. Rev. C60 (1999) 024904, Phys. Lett. B459 (1999) 660.

Phys. Rev. C62 (2000) 064906, nucl-th/0010088.

nucl-th/0010172

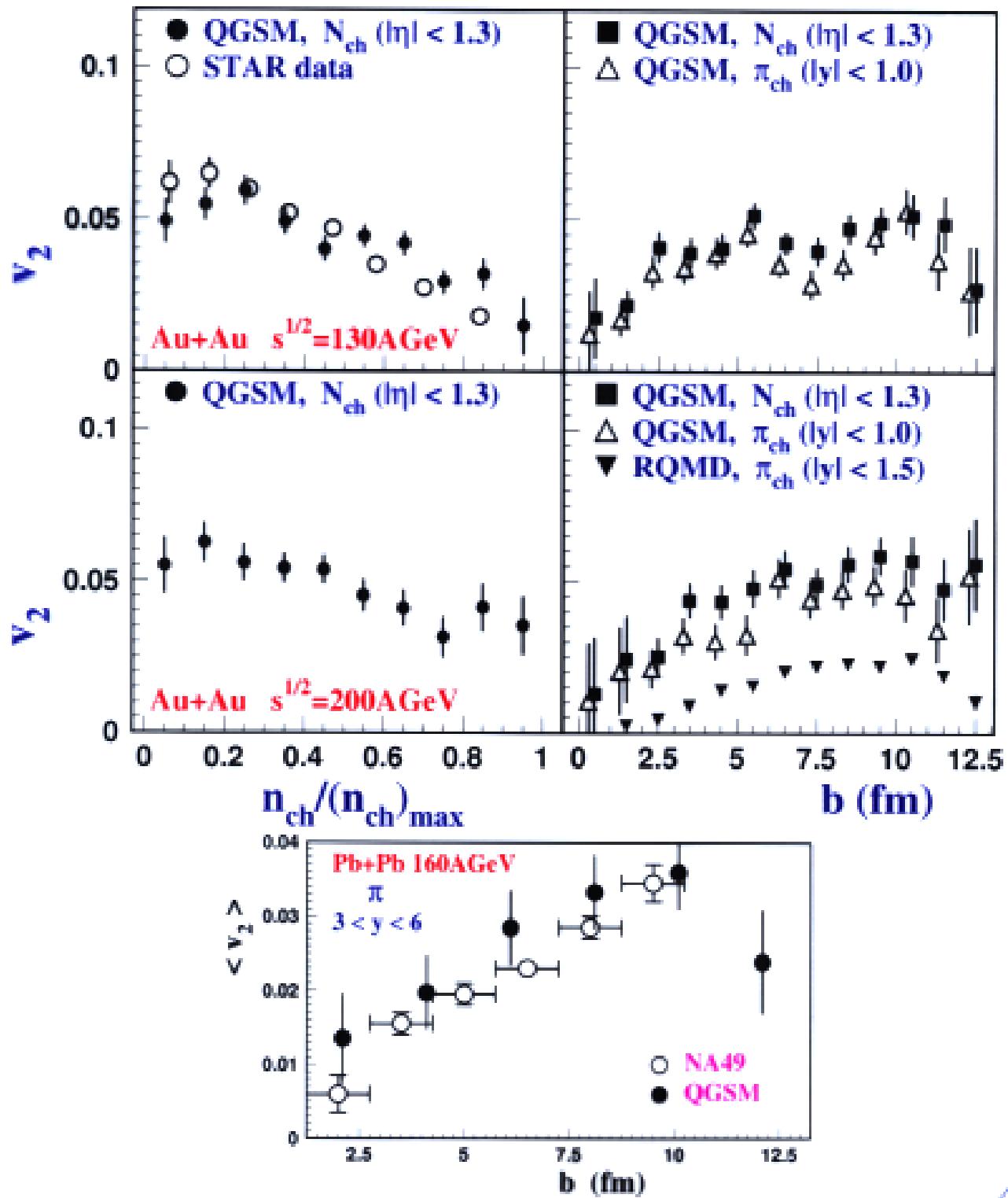
There is a cascade to describe v_2 !

Elliptic flow at SPS and RHIC:

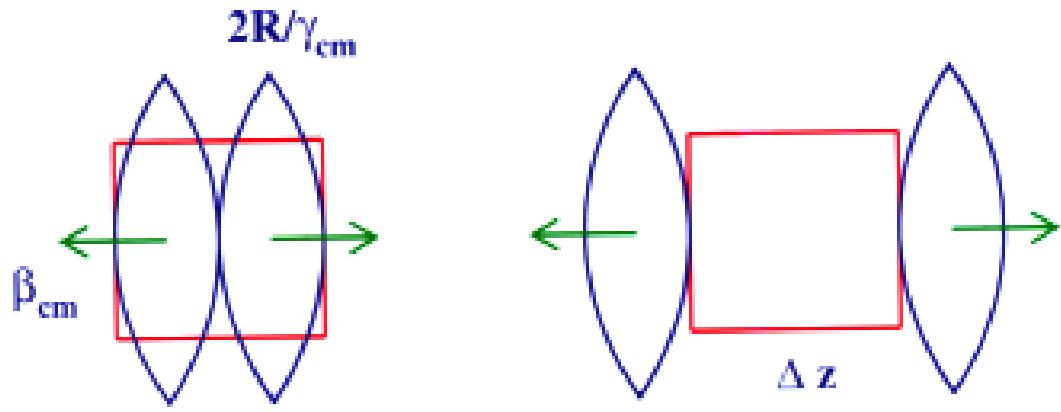
*E. Zabrodin, L. Bravina et al.
in preparation*

K.H. Ackermann et al., STAR Collab., nucl-ex/0009011
R.J.M. Snellings et al., Star Note SN0388 (1999), nucl-ex/9904003

S.A. Voloshin and A.M. Poskanzer, Phys. Lett. B 474, 27 (2000)



Equilibration in the central cell



$$t^{\text{cross}} = 2R/\gamma_{\text{cm}} \beta_{\text{cm}} \quad t^{\text{eq}} \geq t^{\text{cross}} + \Delta z/(2\beta_{\text{cm}})$$

$$t^{\text{cross}} = 5.5 \text{ (AGS), } 1.4 \text{ (SPS), } 0.12 \text{ fm}/c \text{ (RHIC).}$$

KINETIC equilibrium: isotropy of velocity distributions of hadrons
 \Rightarrow isotropy of pressure.

THERMAL equilibrium(elastic collisions): energy spectra of particles are described by Boltzmann distribution with temperature T and chemical potential $\mu_i = B_i \mu_B + S_i \mu_S$:

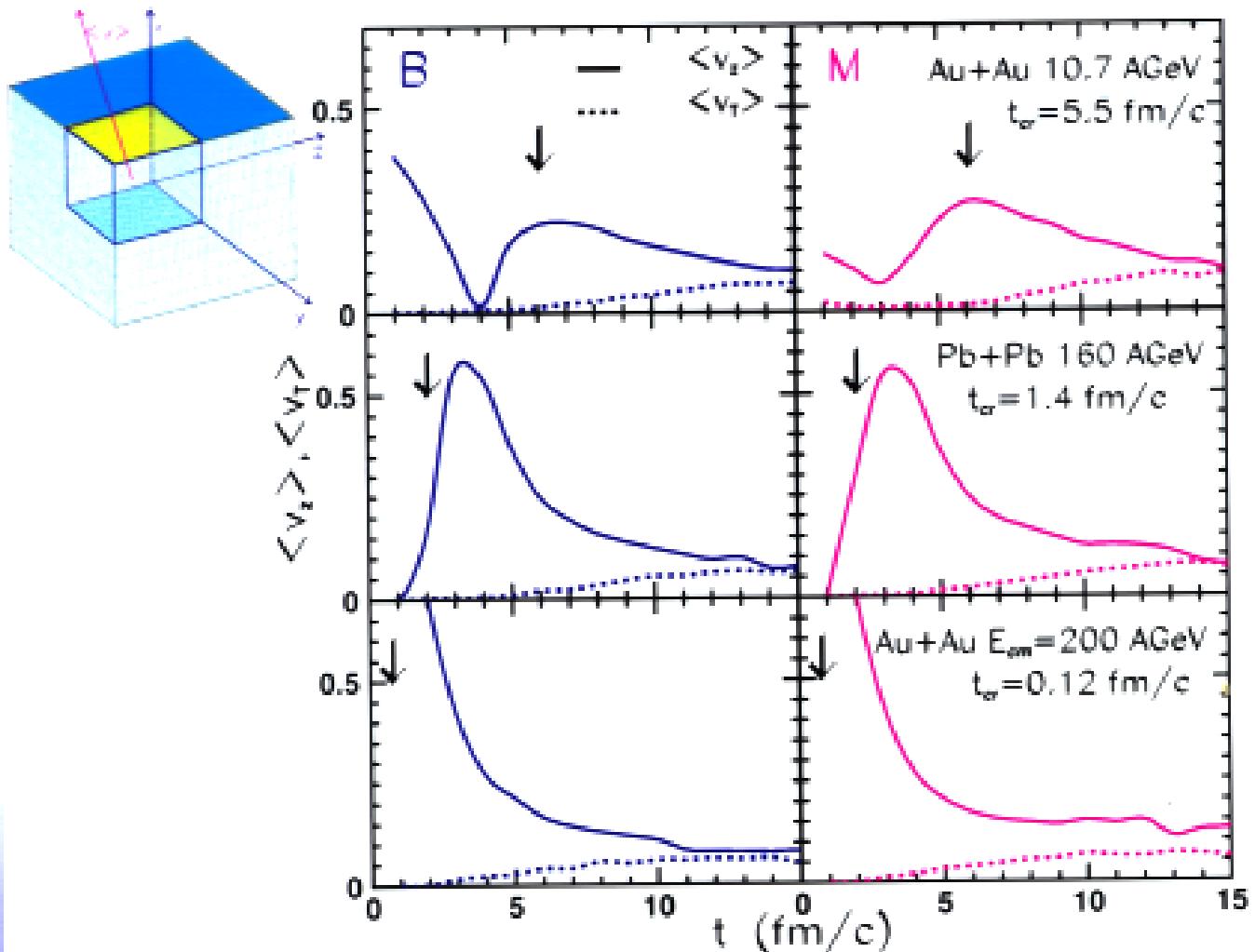
$$\frac{dN_i}{4\pi p E dE} = \frac{V g_i}{(2\pi\hbar)^3} \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

CHEMICAL equilibrium (inelastic collisions and decays of resonances): the yields of particles are reproduced by the SM with the same parameters (T , μ_B , μ_S):

$$N_i = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 dp \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Kinetic equilibrium: Isotropy of the velocity distributions.

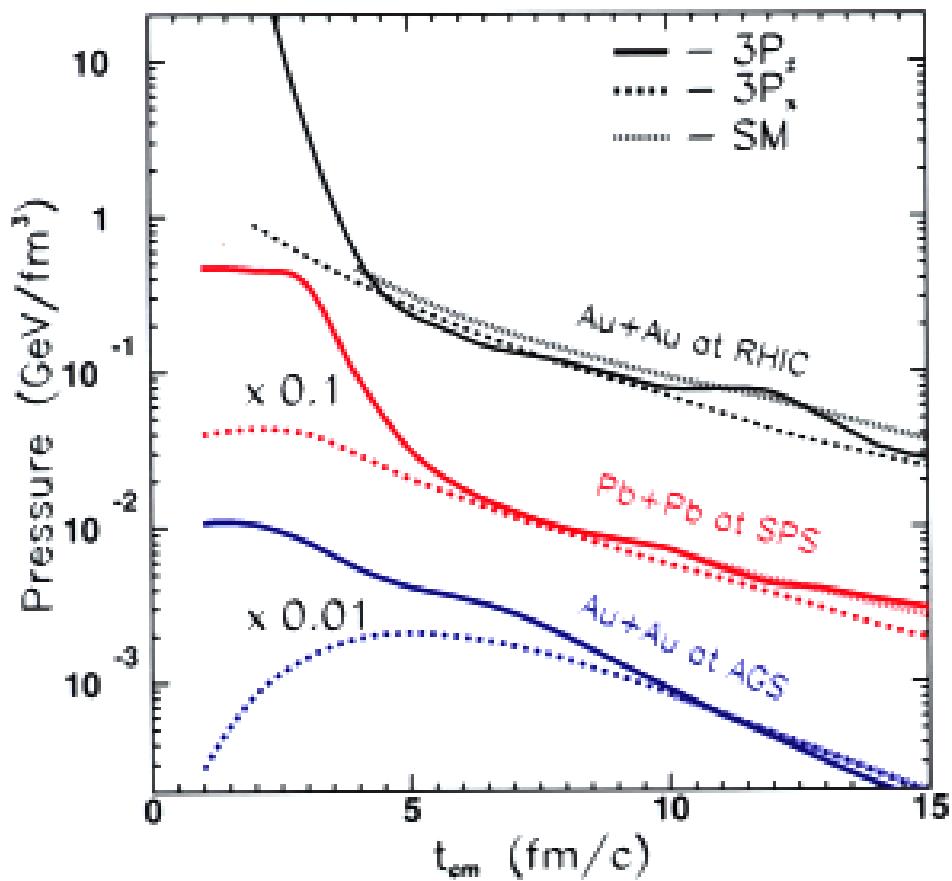
Time evolution of the collective velocities of baryons and mesons in asymmetric cell $0 \leq \{x, y, z\} \leq 2.5$ fm.



RHIC: Velocity distributions become almost isotropic at $t \geq 5$ fm/c.
 Flow is weak, $v_{flow} = 0.1 - 0.15c$ only. This gives small correction,
 $\langle m_N v_{flow}^2 / 2 \rangle \cong 7$ MeV for the nucleon spectra, and less than a MeV
 for pions.

Kinetic equilibrium: Isotropy of the pressure.

The longitudinal ($3P_{\{z\}}$) and the transverse ($3P_{\{x\}}$) diagonal components of the microscopic pressure tensor in the central 125 fm^3 cell of A+A collisions at AGS, SPS, and RHIC. Dotted lines indicate the pressure given by the statistical model.



Pressure becomes isotropic at $t \cong 10 \text{ fm}/c$ (AGS),

$t \cong 7 \text{ fm}/c$ (SPS), and $t \cong 5 \text{ fm}/c$ (RHIC).

Statistical Model of Ideal Hadron Gas

$$\varepsilon^{\text{mic}} = \frac{1}{V} \sum_i E_i^{\text{SM}}(T, \mu_B, \mu_S)$$

$$\rho_B^{\text{mic}} = \frac{1}{V} \sum_i B_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S)$$

$$\rho_S^{\text{mic}} = \frac{1}{V} \sum_i S_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S)$$

$$f(p, m_i) = \exp\left(\frac{\mu_i - E_i}{T}\right), \quad E_i = (p_i^2 + m_i^2)^{1/2}, \quad \mu_i = B_i \mu_B + S_i \mu_S$$

$$n_i^{\text{SM}} = \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 f(p, m_i) dp$$

$$\varepsilon_i^{\text{SM}} = \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 \sqrt{p^2 + m_i^2} f(p, m_i) dp$$

$$P^{\text{SM}} = \sum_i \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 \frac{p^2}{3E_i} f(p, m_i) dp$$

Entropy density can be found via Gibbs thermodynamical identity

$$\varepsilon^{\text{mic}} = T^{\text{SM}} s^{\text{SM}} + \mu_B^{\text{SM}} \rho_B^{\text{mic}} + \mu_S^{\text{SM}} \rho_S^{\text{mic}} - P^{\text{SM}},$$

or via the distribution function $f(p, m_i)$

$$s^{\text{SM}} = - \sum_i \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty f(p, m_i) [\ln f(p, m_i) - 1] p^2 dp$$

Evolution of thermodynamic parameters in the cell

RHIC: Au+Au at $\sqrt{s} = 200 \text{ AGeV}$, $t = 5 - 18 \text{ fm/c}$.

The temperature T^{SM} , baryon chemical potential μ_B^{SM} , strange chemical potential μ_S^{SM} , pressure P , and entropy per baryon s/ρ_B , are extracted from the SM, using the energy density $\varepsilon^{\text{cell}}$, baryonic density ρ_B^{cell} , and strangeness density ρ_S^{cell} as input.

time fm/c	$\varepsilon^{\text{cell}}$ MeV/fm ³	ρ_B^{cell} fm ⁻³	ρ_S^{cell} fm ⁻³	T^{SM} MeV	μ_B^{SM} MeV	μ_S^{SM} MeV	P MeV/fm ³	s/ρ_B^{cell}
5	2330	0.093	-0.0042	201	39.7	13.0	349	143
6	1705	0.071	-0.0047	193	41.0	12.1	257	142
7	1319	0.059	-0.0011	187	44.2	13.4	201	138
8	1031	0.045	0.0011	181	44.3	13.6	159	144
9	820	0.040	-0.0044	176	47.2	10.4	128	135
10	656	0.029	-0.0022	171	44.1	9.8	104	152
11	544	0.025	-0.0044	167	47.4	11.2	87	149
12	446	0.018	0.0089	163	42.6	10.6	72	176
13	346	0.015	0.0040	158	50.4	15.6	57	165
14	290	0.012	0.0024	154	48.0	12.9	49	180
15	241	0.009	-0.0044	150	40.7	6.5	41	214
16	200	0.007	0.0022	147	37.8	6.6	35	250
17	168	0.006	-0.0031	143	42.7	-1.1	30	218
18	145	0.005	-0.0028	140	43.4	-1.9	26	227

Expansion proceeds with constant and small $\mu_B \approx 45 \text{ MeV}$
 T varies from 201 MeV to 140 MeV

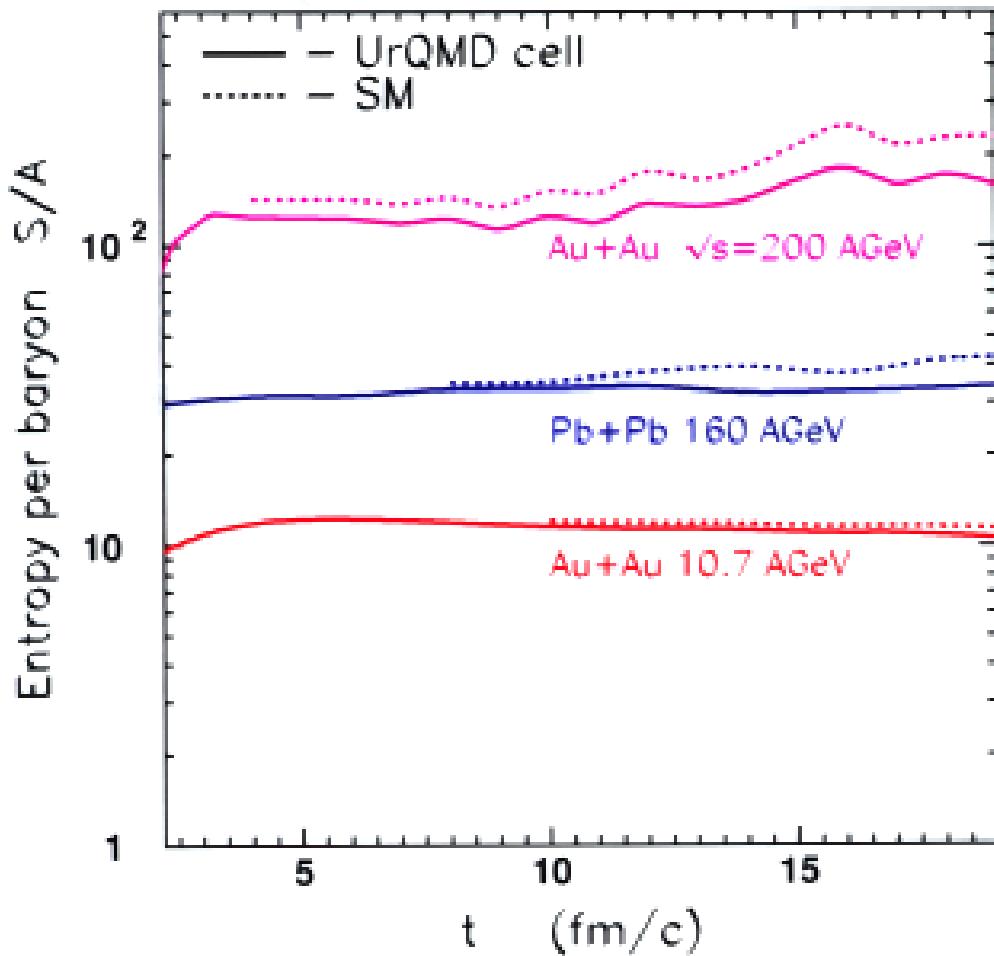
How far is the SM equilibrium? - Entropy analysis

Entropy density:

$$s^{SM} = T^{-1} (\varepsilon - \mu_B \rho_B - \mu_S \rho_S + P) ,$$

$$s^{mic} = - \sum_i \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty f_i(p, m_i) [\ln f_i(p, m_i) - 1] p^2 dp$$

Time evolution of the ratio $S/A \equiv s/\rho_B$ in the central cell in A+A collisions at AGS, SPS, and RHIC energies.



The entropy per baryon ratio stays almost constant.

$S/A = 12$ (AGS), 32 (SPS), 150 ± 10 (RHIC).

Fit to experimental data gives $S/A = 14$ (AGS) and 36 (SPS) !!

Cleymans&Redlich, Phys. Rev. C60 (1999) 054908

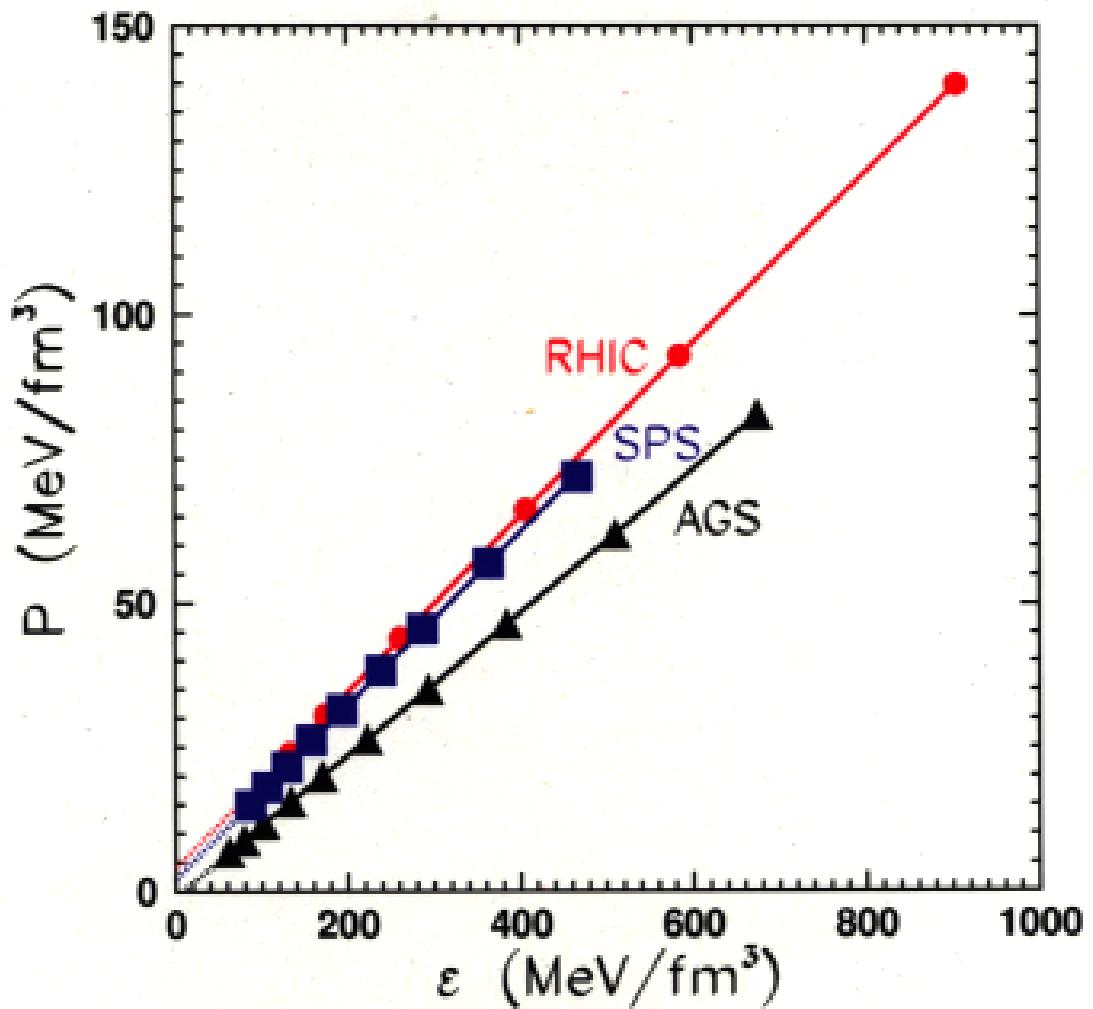
Equation of state: P vs ε

All particles in the central cell in A+A collisions.

AGS: Au+Au at 10.7 AGeV, $t = 10 - 19$ fm/c

SPS: Pb+Pb at 160 AGeV, $t = 10 - 19$ fm/c

RHIC: Au+Au at $\sqrt{s} = 200$ AGeV, $t = 12 - 19$ fm/c



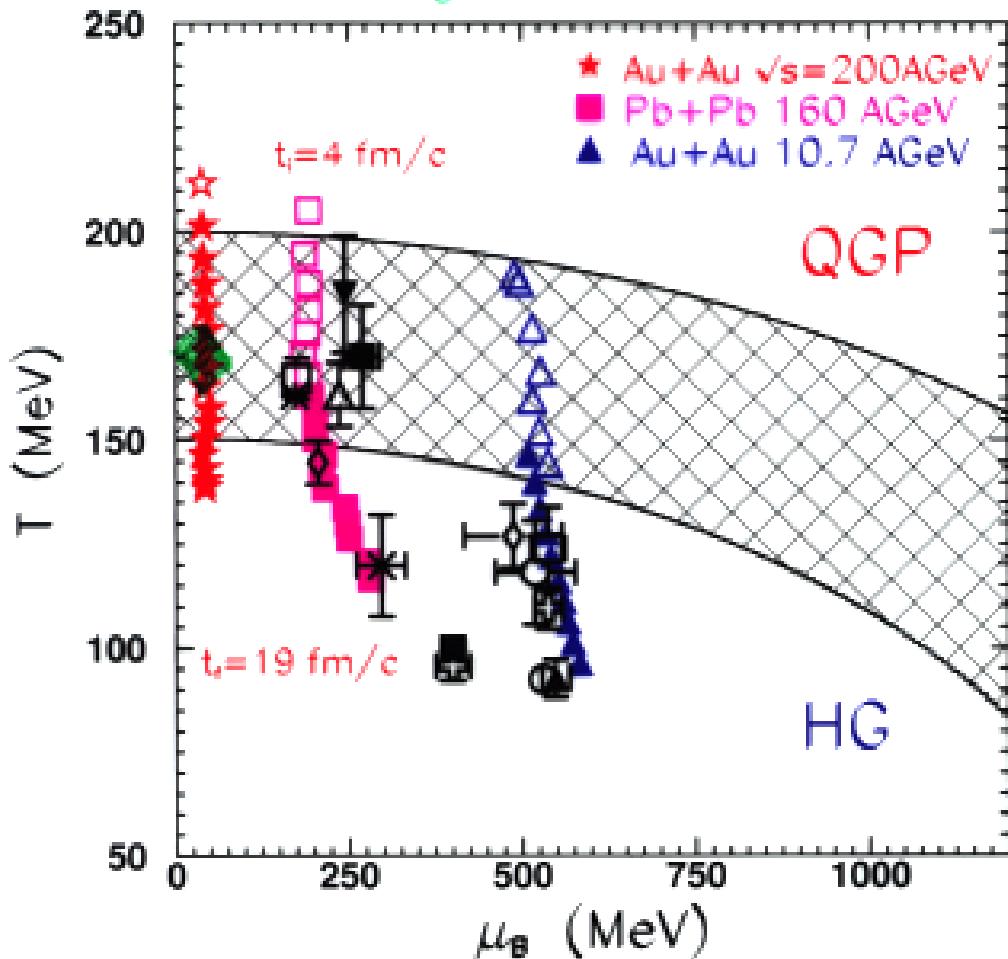
Expansion proceeds at constant P/ε ratio.

$P/\varepsilon = 0.12$ (AGS), 0.15 (SPS), 0.15 (RHIC).

Equation of state: T vs μ_B

Comparison with chemical freeze-out (**upper points**) and thermal freeze-out (**lower points**) parameters at AGS and SPS energies (compilation from J. Cleymans and K. Redlich, Phys. Rev. C60 (1999) 054908)

 - new RHIC data analysis



MIT Bag model Boundary of the **QGP** for gluons and massless quarks.

$B^{1/4} = 227$ and 302 MeV ($T_c = 150$ and 200 MeV)

$$B = P_Q + P_G = T_c^4 \left(\frac{95}{180\pi^2} + \frac{1}{9} \left(\frac{\mu_B}{T_c} \right)^2 + \frac{1}{(162\pi^2)} \left(\frac{\mu_B}{T_c} \right)^4 \right)$$

AGS and SPS: Chemical and thermal freeze-out conditions are close to initial and final conditions of the kinetic equilibrium stage in the cell

Energy per particle in the cell

Phenomenological observation: all freeze-out parameters correspond to an average energy per hadron of ≈ 1 GeV
 (J. Cleymans and K. Redlich, Nucl. Phys. A661 (1999) 379c).

Energy per particle in the cell at the beginning of kinetic equilibrium:

	time fm/c	ε GeV/fm ³	$\langle E \rangle / \langle N \rangle$ MeV/particle
AGS	10	0.675	1.03
SPS	10	0.468	0.91
RHIC	5	2.330	1.24
RHIC	7	1.319	1.08
RHIC	9	0.820	0.90

Applying the criterion $\langle E \rangle / \langle N \rangle \approx 1$ GeV, we get the most probable values of the temperature and baryon chemical potential at chemical freeze-out in Au+Au at RHIC ($\sqrt{s} = 200$ AGeV):

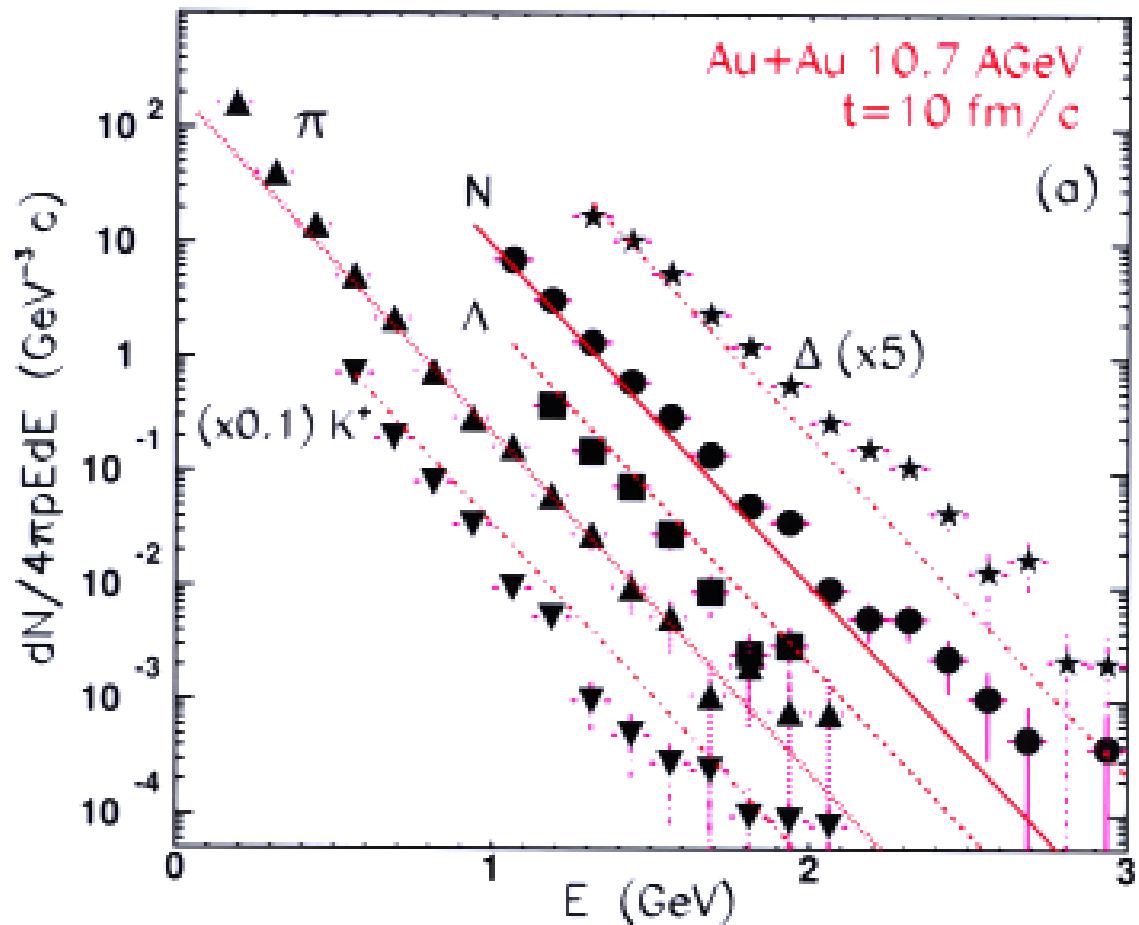
new RHIC data

$$T = 175 \pm 5 \text{ MeV} ; \quad 170 \text{ MeV}$$

$$\mu_B = 45 \pm 4 \text{ MeV} \quad \sim 50 \text{ MeV}$$

(Redlich)
today talk

Thermalization: Energy spectra at AGS



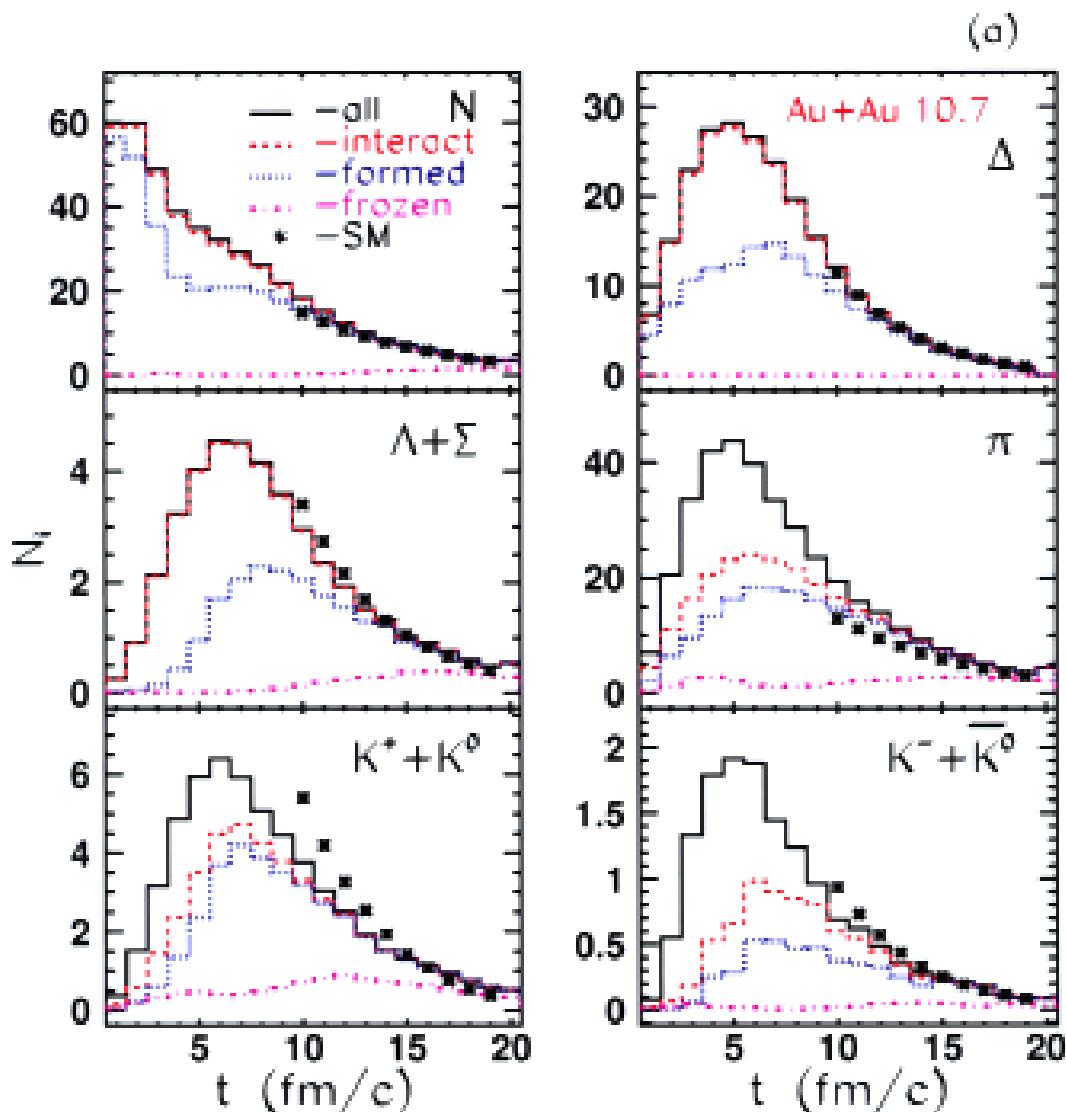
All particles in the central cell at $t = 10 \text{ fm}/c$.

Fit by the SM with the parameters:

$T = 147 \text{ MeV}$; $\mu_B = 510 \text{ MeV}$; $\mu_S = 128 \text{ MeV}$

Thermal equilibrium seems to be reached.

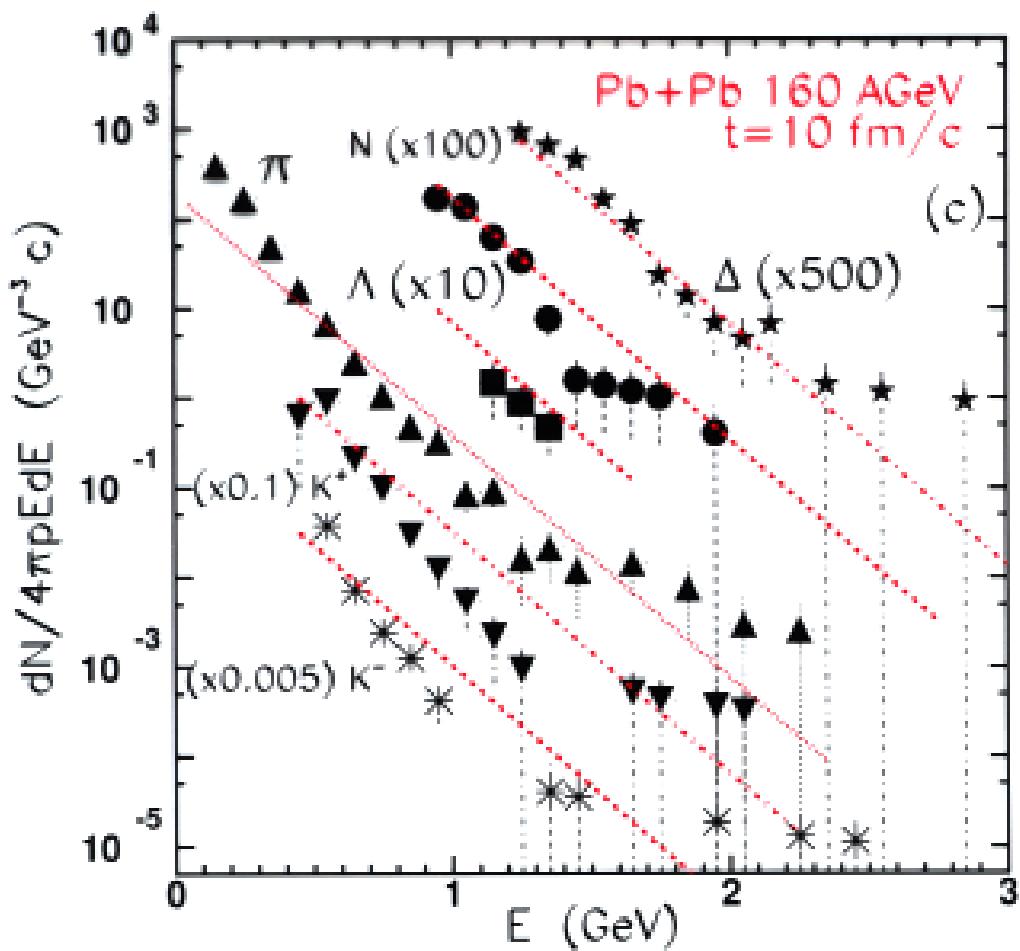
Relaxation to chemical equilibrium: yields at AGS



All particles in the central cell at $10 \leq t \leq 18$ fm/c.

Chemical equilibrium seems to be reached.

Thermalization: energy spectra at SPS



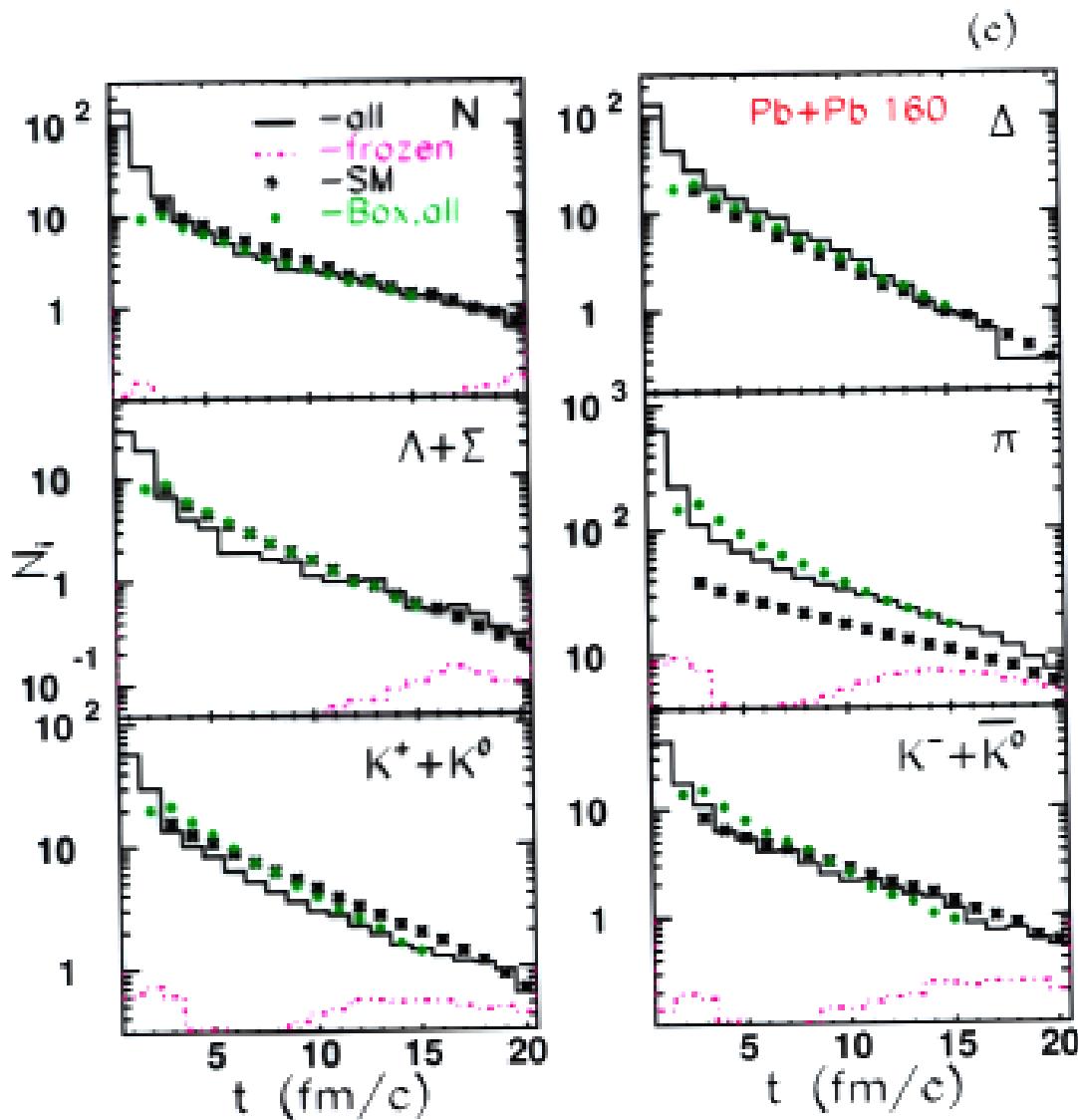
All particles in the central cell at $t = 10 \text{ fm}/c$.

Fit by the SM with the parameters:

$T = 161 \text{ MeV}$; $\mu_B = 197 \text{ MeV}$; $\mu_S = 37 \text{ MeV}$

Mesons seem to have lower temperature compared to baryons.

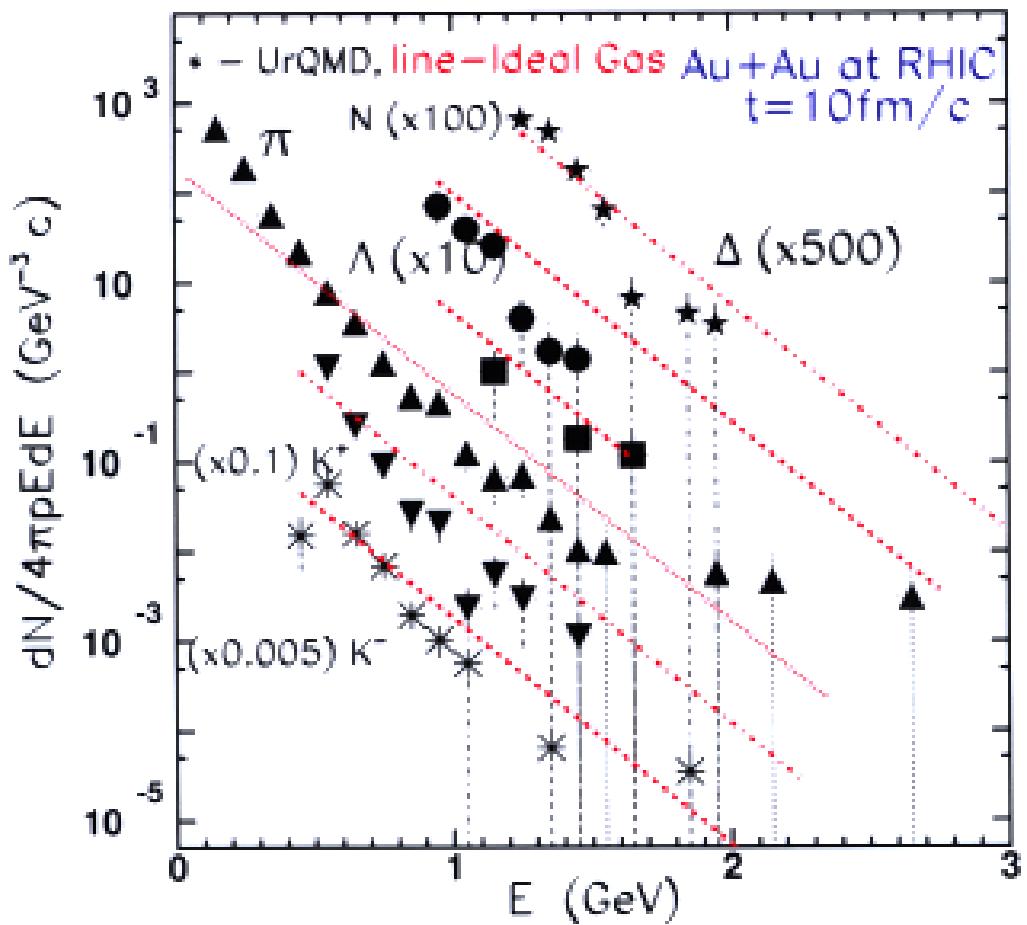
Relaxation to chemical equilibrium: yields at SPS



All particles in the central cell at $10 \leq t \leq 18$ fm/c.

Baryons and kaons are in good agreement, pions are strongly underpredicted by the SM.

Thermalization: energy spectra at RHIC



Parameters of the SM:

$$T = 171 \text{ MeV}; \mu_B = 44 \text{ MeV}; \mu_S = 10 \text{ MeV}$$

Exponential fit to energy spectra gives:

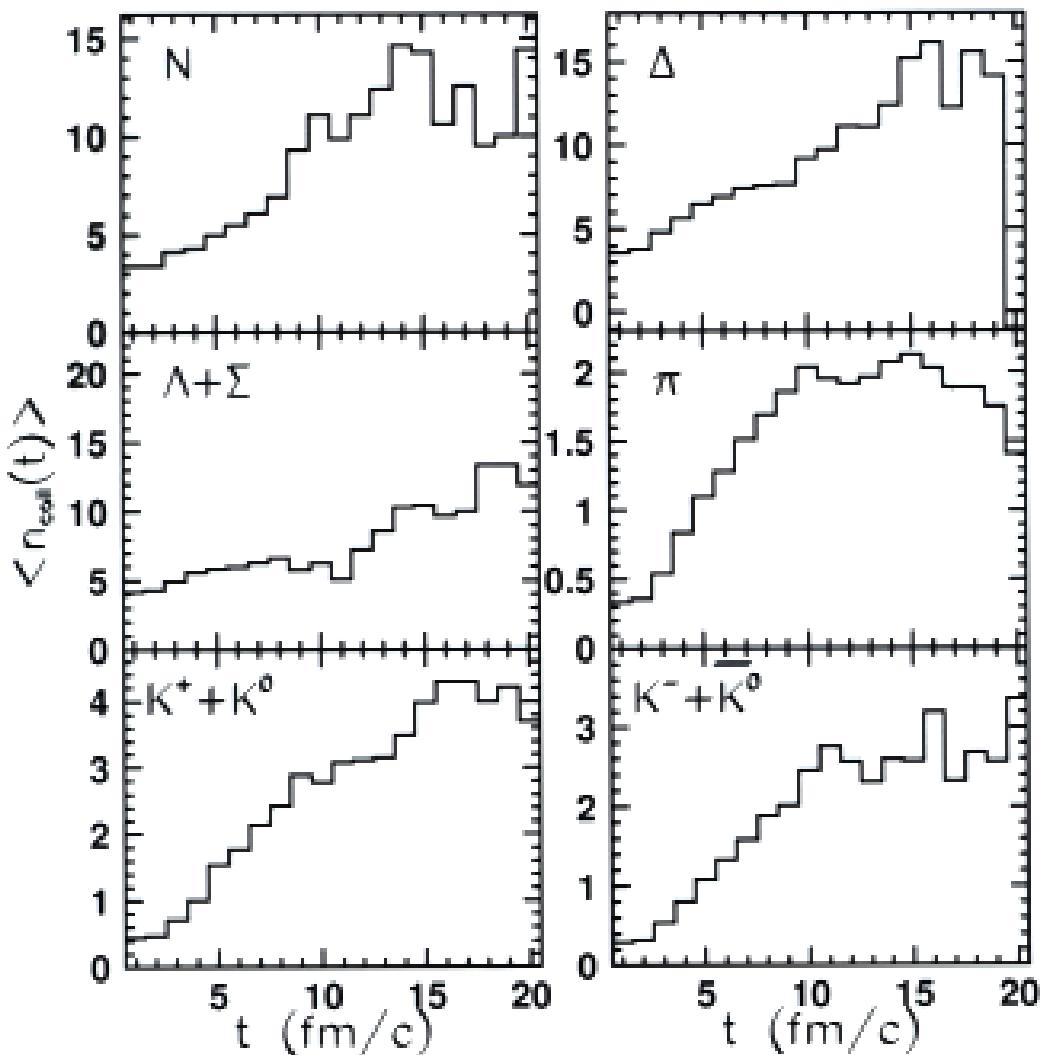
$$T^N = 113 \text{ MeV};$$

$$T_{(1)}^\pi = 96 \text{ MeV}, T_{(2)}^\pi = 160 \text{ MeV} \text{ (two slopes)};$$

$$T^K = 85 \text{ MeV}$$

Mesons have lower temperatures than baryons

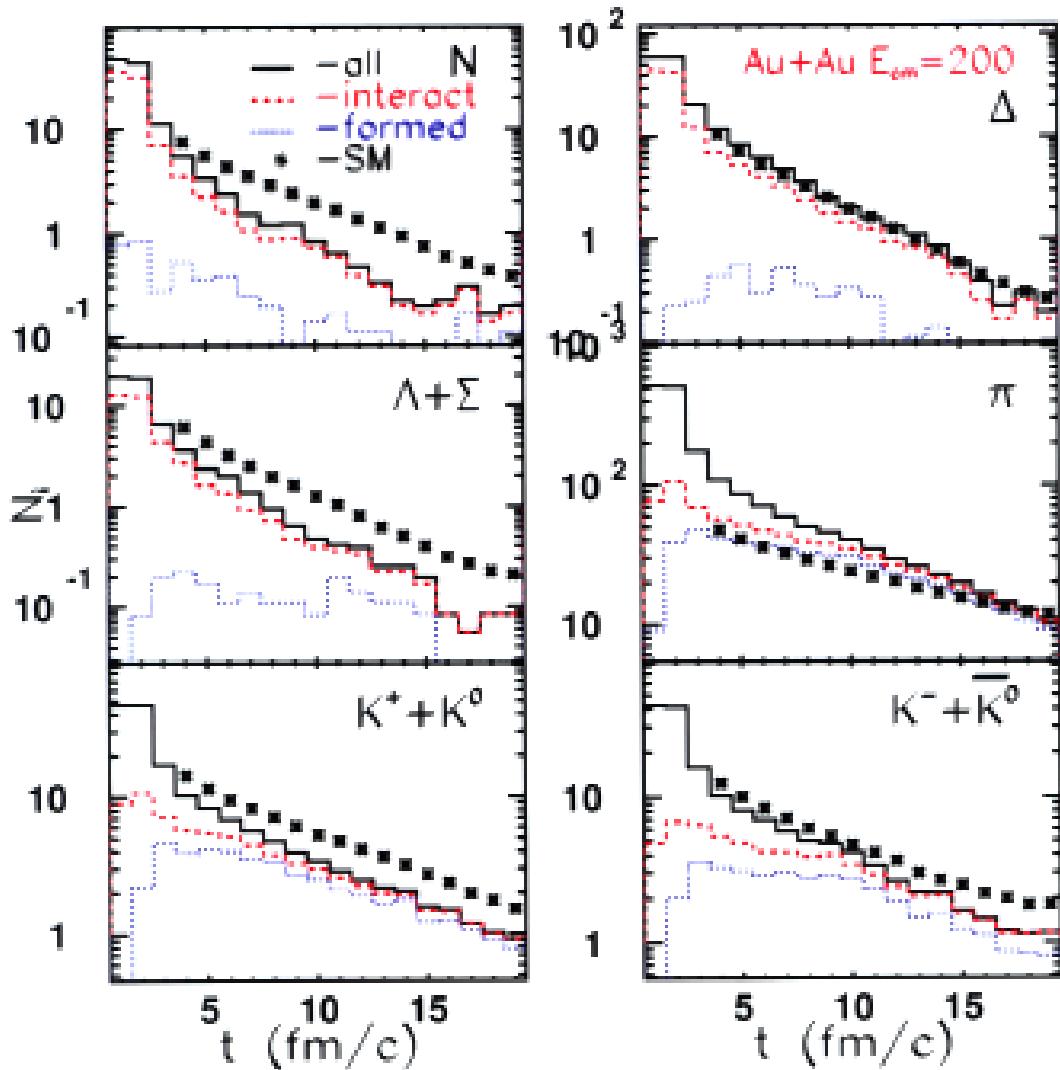
Collision rates in the central cell at RHIC



Number of elastic collisions per pion, 1 at $t=5$ fm/c and 2 at $t=10$ fm/c, is not enough to reach thermal equilibrium.

Significant part of pions is coming from the decay of resonances or inelastic scattering (non-thermalized processes).

Relaxation to chemical equilibrium: yields at RHIC



Particles in the central cell at $5 \leq t \leq 19$ fm/c.

Only Δ 's are fitted nicely.

Since hypercharge $Y = B + S$ is conserved in strong interactions, maybe the rest of the Y is in resonances?!

Formation of resonance-rich matter at RHIC

At RHIC the meson-dominated matter is produced.

	Mesons $(N_M/N_{tot})^{cell}$	Baryons $(N_B/N_{tot})^{cell}$	Antibaryons $(N_{\bar{B}}/N_{tot})^{cell}$
RHIC	90%	7%	3%
SPS	85%	14.5%	0.5%
AGS	50%	50%	0%

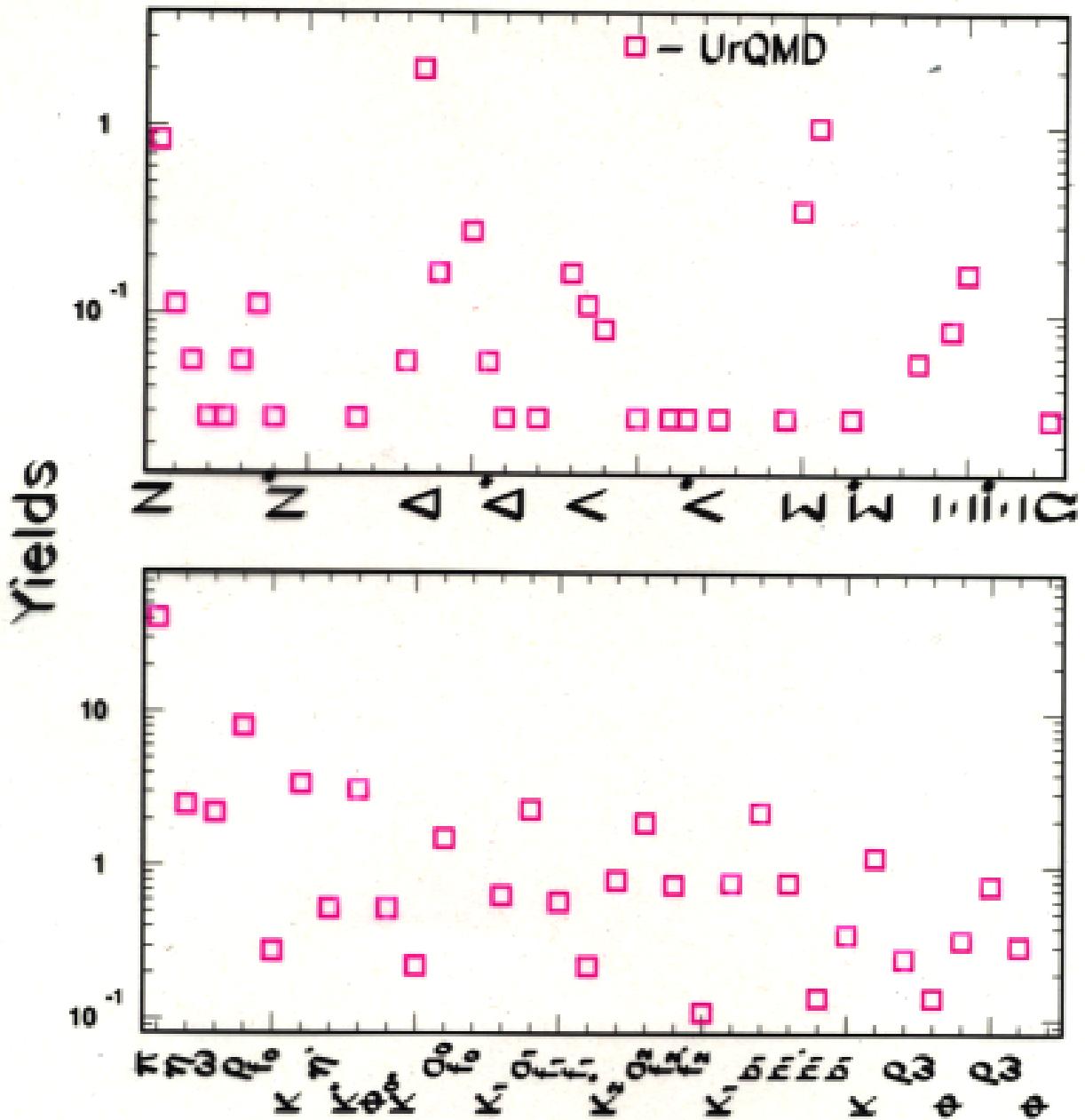
At RHIC energies the admixture of antibaryons is significant.

Fraction of resonances:

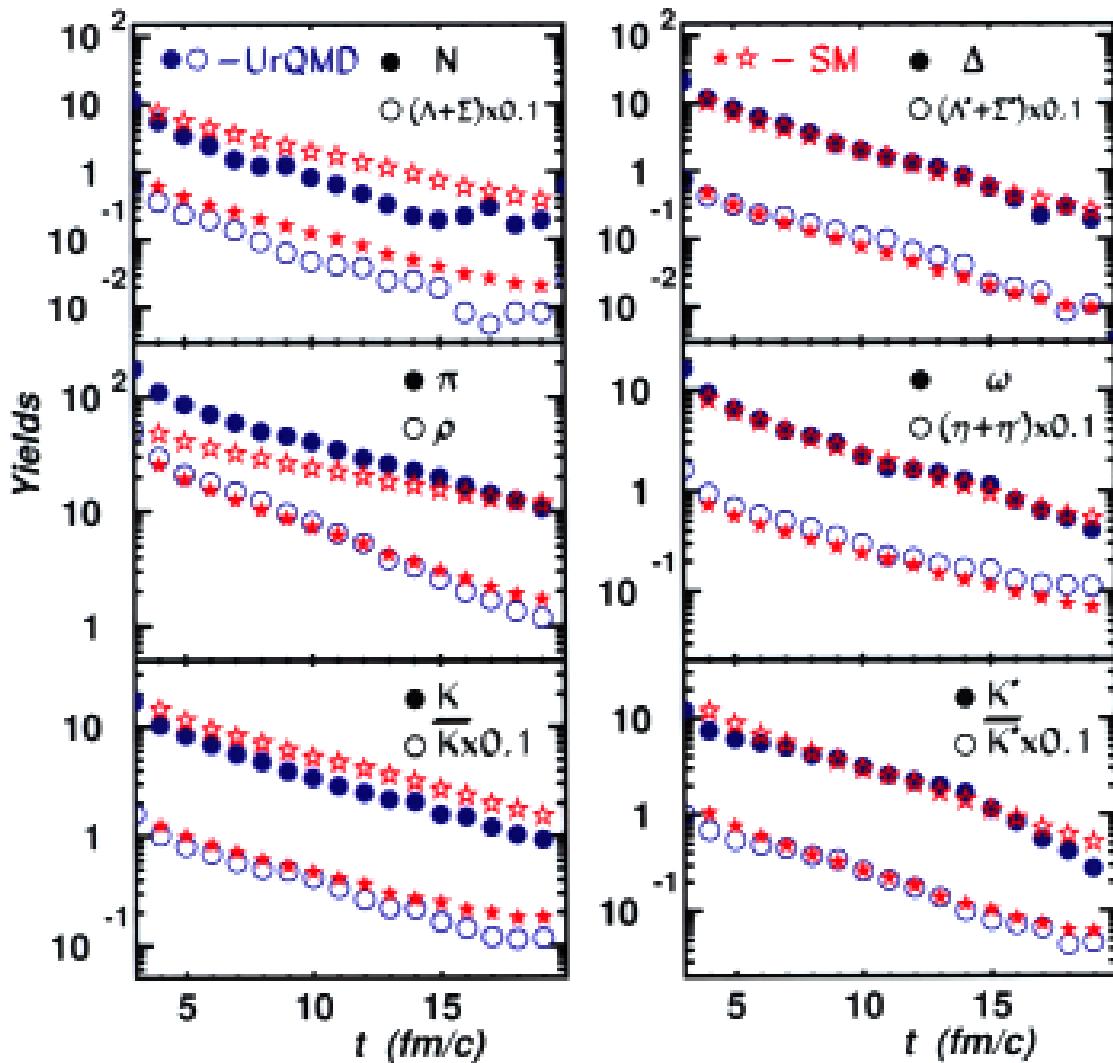
	Mesons (5 fm/c \rightarrow 20 fm/c) $(N_R^M/N_{tot}^M)^{cell}$	Baryons (5 fm/c \rightarrow 20 fm/c) $(N_R^B/N_{tot}^B)^{cell}$
RHIC	60% \rightarrow 30%	70% \rightarrow 70%
SPS	50% \rightarrow 20%	70% \rightarrow 35%
AGS	40% \rightarrow 15%	60% \rightarrow 25%

RHIC: fraction of resonances dominates up to $t \approx 20$ fm/c, i.e. resonance-rich matter is survived almost to the freeze-out.

Hadron species in the cell at RHIC at $t=10$



Yields in the cell at RHIC



Particles in the central cell at $5 \leq t \leq 19$ fm/c.

Both strange and non-strange resonances are well matched to the SM predictions. Pions from $t \leq 14$ fm/c are underestimated, other particles are overestimated by the SM.

Where is the rest of the hypercharge??

Net and partial baryon densities in the cell at RHIC

$$\rho_B^{net} \equiv R_B - R_{\bar{B}}$$

The partial densities of B 's and \bar{B} 's, $R_{B(\bar{B})}$, from the time interval $5 \leq t \leq 15$ fm/c in the central cell of Au+Au collisions at RHIC.

time fm/c	R_B^{mic} fm^{-3}	$R_{\bar{B}}^{mic}$ fm^{-3}	R_B^{SM} fm^{-3}	$R_{\bar{B}}^{SM}$ fm^{-3}
5	0.183	0.090	0.332	0.239
6	0.135	0.064	0.236	0.165
7	0.106	0.047	0.178	0.119
8	0.082	0.036	0.134	0.088
9	0.065	0.025	0.105	0.065
10	0.049	0.020	0.079	0.050
11	0.041	0.016	0.064	0.039
12	0.031	0.013	0.049	0.031
13	0.025	0.010	0.036	0.021
14	0.020	0.008	0.029	0.017
15	0.016	0.007	0.022	0.013

Net baryon densities are equal, $\rho_B^{mic} = \rho_B^{SM}$,

but partial densities are not: $R_B^{SM} > R_B^{mic}$; $R_{\bar{B}}^{SM} > R_{\bar{B}}^{mic}$

Net and partial strangeness densities in the cell at RHIC

$$\rho_S^{net} \equiv (R_S^B - R_S^{\bar{B}}) + (R_S^M - R_S^{\bar{M}})$$

The partial strangeness densities of B 's, M 's, and their antiparticles, $R_S^{(B,\bar{B},M,\bar{M})}$, from the time interval $5 \leq t \leq 10$ fm/c in the central cell of Au+Au collisions at RHIC.

time fm/c	(R_S^B) fm^{-3}	$(R_S^{\bar{B}})$ fm^{-3}	(R_S^M) fm^{-3}	$(R_S^{\bar{M}})$ fm^{-3}
5 UrQMD	-0.064	-0.039	0.216	0.194
	SM	-0.160	-0.129	0.216
6 UrQMD	-0.049	-0.029	0.169	0.154
	SM	-0.112	-0.088	0.169
7 UrQMD	-0.038	-0.020	0.141	0.124
	SM	-0.083	-0.063	0.139
8 UrQMD	-0.031	-0.017	0.115	0.099
	SM	-0.062	-0.047	0.115
9 UrQMD	-0.024	-0.013	0.094	0.087
	SM	-0.048	-0.033	0.094
10 UrQMD	-0.019	-0.010	0.077	0.071
	SM	-0.036	-0.025	0.078

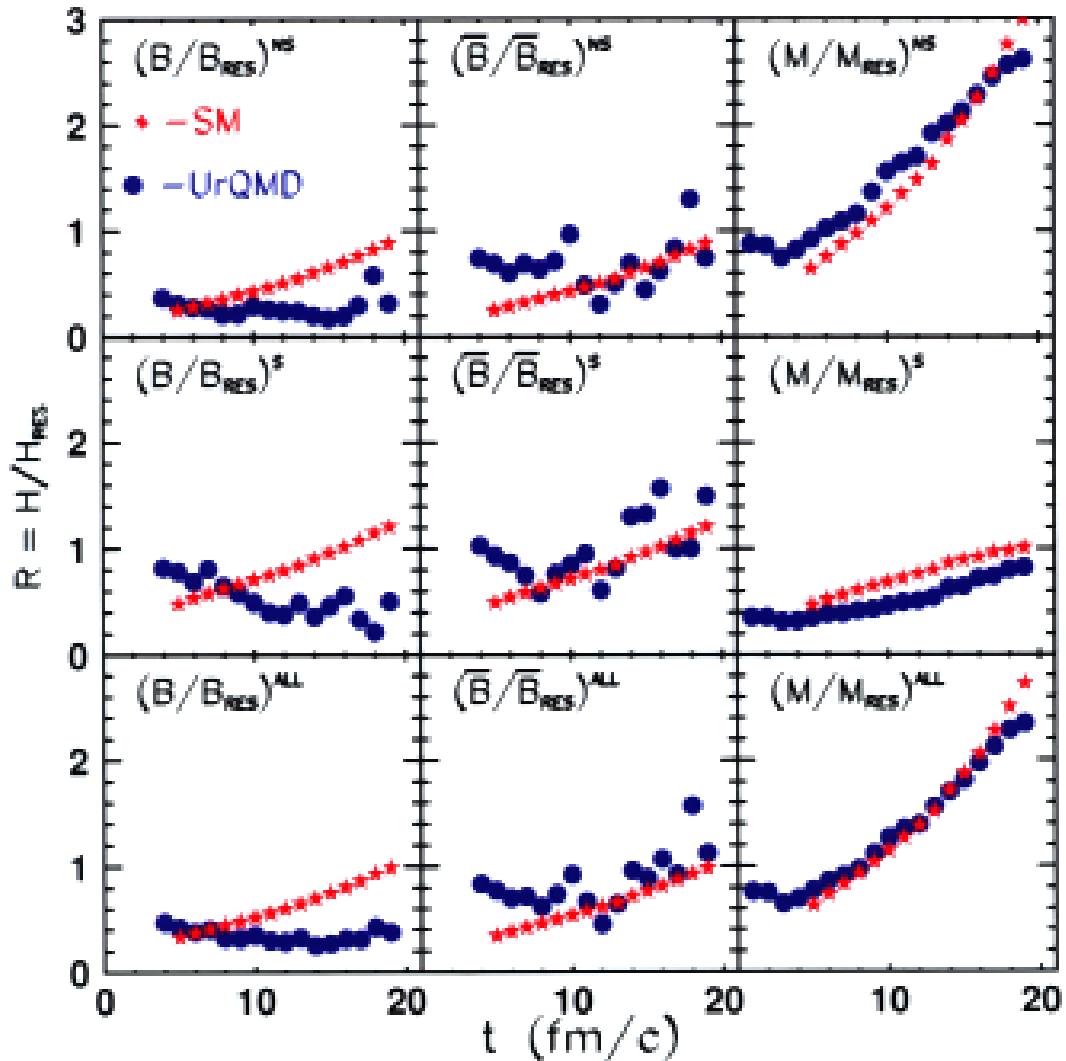
Net strangeness densities are equal, $\rho_S^{mic} = \rho_S^{SM}$,

but partial densities are not: $(R_S^B)^{SM} > (R_S^B)^{mic}$;

$(R_S^{\bar{B}})^{SM} > (R_S^{\bar{B}})^{mic}$; $(R_S^M)^{SM} = (R_S^M)^{mic} !!$

Ratios of hadronic abundances in the cell at RHIC

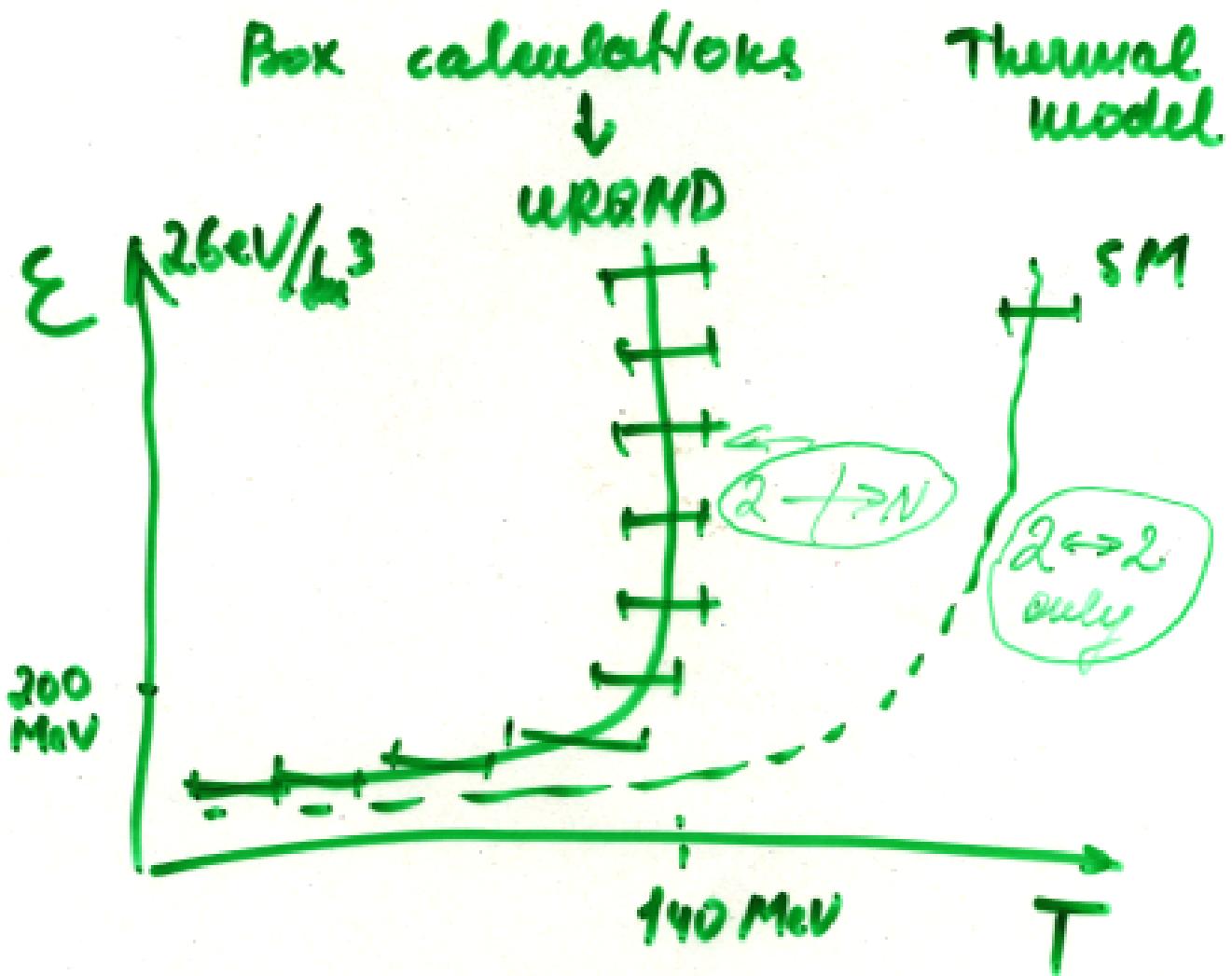
Particles in the central cell at $5 \leq t \leq 19$ fm/c.



Ratio $(M/M_{res})^{mic} \cong (M/M_{res})^{SM}$ at $5 \leq t \leq 18$ fm/c
 (NB: $N_{\pi}^{mic} > N_{\pi}^{SM}$!!);

Ratio $(B/B_{res})^{mic}$ is almost constant due to formation of long-lived resonance-rich matter.

Infinite matter



Violation of detailed balance

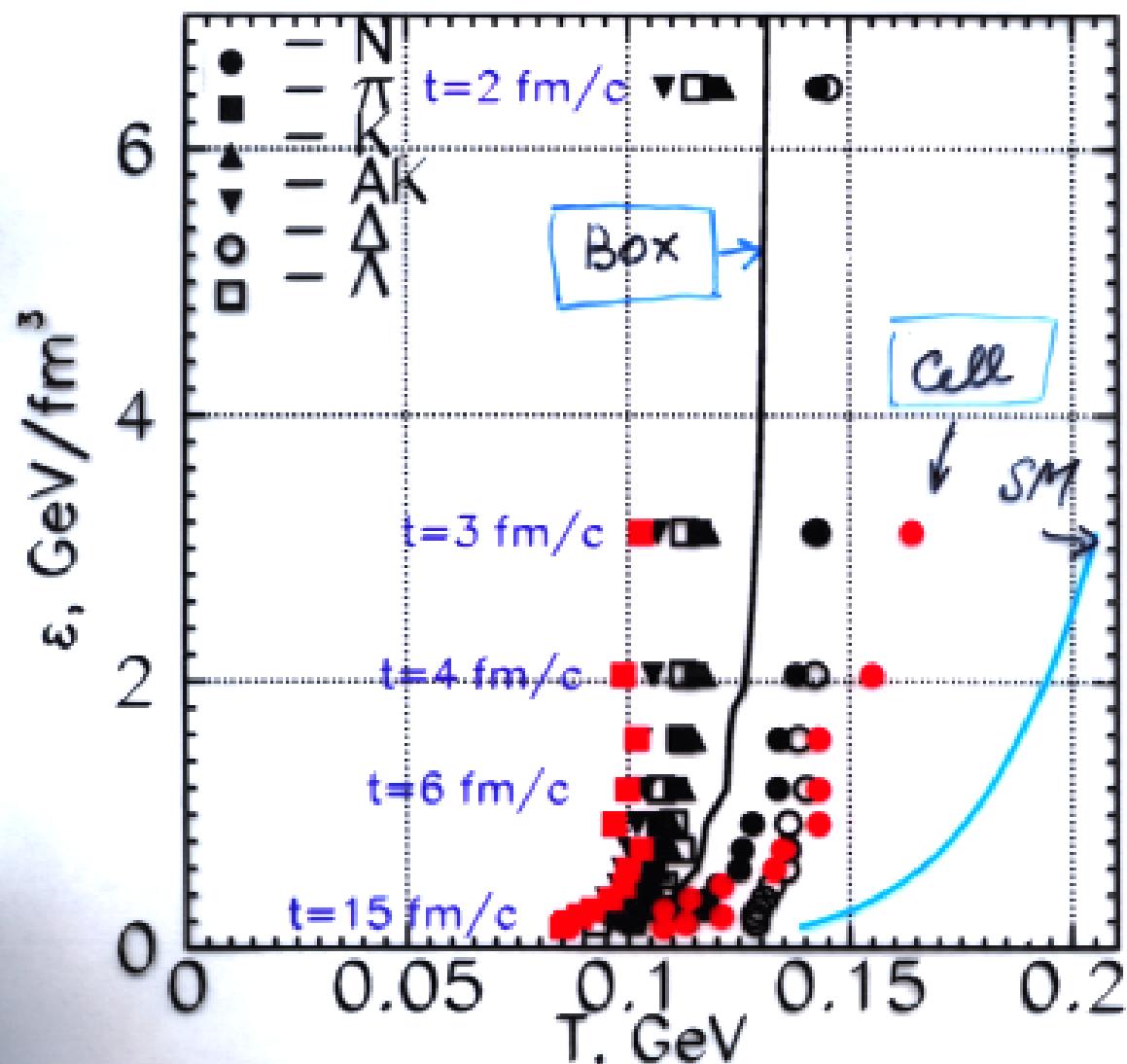
$2 \rightarrow N$

changes EoS to saturated T EoS

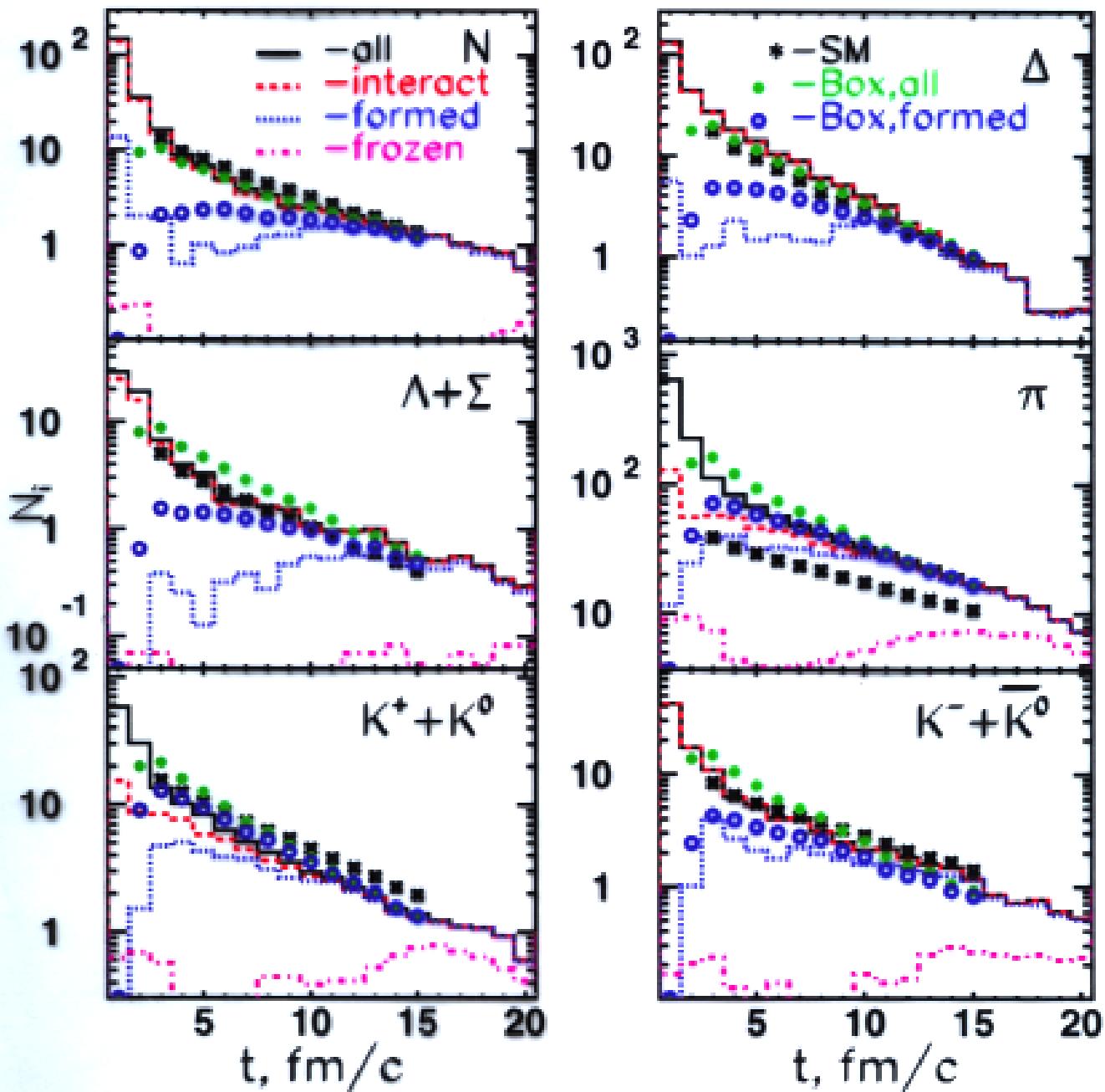
EoS in Box: M. Bellescu et al Phys Rev C 58 (1998) 4722
 EoS in cell L. Bravina et al J. Phys G 25 (1999) 353
 comparison with box L. Bravina et al submitted to Phys Rev C (1999)

Equation of State for Pb+Pb ($b=0$) at 160 AGeV

- box, - fit Pb+Pb-cell, - Ideal Gas



Central cell in Pb+Pb ($b=0$) at 160 AGeV/c. Fit by Statistical Model



Conclusions

- There is a kinetic equilibrium stage (isotropy of the pressure, weak collective flow) of hadron-resonance-string matter in the central cell at RHIC at about $5 \text{ fm}/c \leq t \leq 19 \text{ fm}/c$.
- The ratio P/ε is approximately constant and equals 0.12 (AGS), 0.15 (SPS), and 0.15 (RHIC).
- Entropy per baryon ratio, $S/A = 150 \pm 10$ varies slightly during the time interval $5 \text{ fm}/c \leq t \leq 19 \text{ fm}/c$. Predictions for Au+Au at RHIC (full volume): $S/A \cong 170$.
- AGS and SPS: Conditions of chemical and thermal freeze-out are close to initial and final conditions for the kinetic equilibrium in the cell;
Predictions for Au+Au at RHIC :
 $T \approx 175 \text{ MeV}$; $\mu_B \approx 45 \text{ MeV}$.
- Creation of long-lived resonance-abundant matter decelerates the relaxation to chemical equilibrium; time $t \cong 20 \text{ fm}/c$ is, probably, too short to reach fully equilibrated state.
- Both strange and non-strange resonance-rich matter survives until the thermal freeze-out and can be detected experimentally.